Low-Lying Positive Parity States of Ne^{21}†

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(Received May 14, 1963; revised manuscript received 1 August 1963)

The energy spectrum of Ne^{21} below 6 MeV and some ground-state and excited-state properties are discussed in terms of a model implying the coupling of a *s-d* particle to a rotational Ne^{20} core. Fair agreement with the experimental data is obtained. A comparison with the standard collective model calculations is given.

1. INTRODUCTION

THE model employed here for interpretation of the positive parity states of Ne^{21} below 6 MeV is essentially a version of the "core-particle coupling" model. The idea of coupling an outside nucleon to a core is extensively used within the framework of the collective and shell models. We find an interesting remark on the concept of a "core" in a paper by Lane.[1] He calculates the positive parity states of N^{13} and C^{13} by coupling a $2s-1d$ particle to the "parent" configuration $(1p)^8$ of C^{12} and antisymmetrizing the wave functions of the combined systems properly. He states that even though the $2s-1p$ and $1d-1p$ interaction integrals are as large as the $1p-1p$ interaction integral, the coupling of the extra particle does not seem to disturb the core appreciably, as is confirmed by his results. Thus, the picture of the core does not seem to be crucially affected by the addition of a particle.

Litherland *et al.*[2] have shown that the spectrum of Ne^{20} can be resolved into various rotational bands. The coupled system of the positive parity ground-state band of Ne^{20} and a $2s-1d$ neutron should represent the low-lying and some of the higher positive parity states of Ne^{21} with isotopic spin $T=\frac{1}{2}$. The Hamiltonian of the total system $Ne^{21}=Ne^{20}+$neutron takes the form

$$H=H_{\text{core}}+H_{\text{neutron}}+H_{\text{coupling}}. \tag{1.1}$$

For the coupling of the outside particle and the core we will use an interaction term similar to the one employed in the collective model:

$$H_{\text{coll. coupling}}=-f(r_p)\textstyle\sum_k \alpha_k{}^c Y_k(\vartheta_p,\varphi_p). \tag{1.2}$$

Usually only the term with $k=2$ is taken into consideration. Furthermore, an extension of the discussion of the spin-orbit force in the framework of the optical model leads to the introduction of a coupling term between the particle spin and the angular momentum of the core of opposite sign as the spin-orbit coupling.

By use of a rotational wave function for the core instead of a much more complicated four particle wave function, we gain much in simplicity of calculation; but, as in any collective model calculation of this type, we lose the possibility of antisymmetrizing our coupled wave functions with respect to exchange of the additional particle and the core particles, for we do not know the dependence of the collective coordinates on the single-particle coordinates. The validity of this approximation has not been examined in detail so far.[3]

If we consider the number of levels obtained by applying the SU_3 classification scheme[4] to Ne^{21}, we can at least give some crude improvements on this situation. Assuming the Ne^{20} ground-state band to be of the spatial structure

$$[4],\quad (8,0) \text{ with } l=0,2,4,6,8 \quad (K=0);$$

([f] partition of the number of particles, corresponds to multiplet classification; (λ,μ) partition of irreducible representation of SU_3), we find that the addition of another $s-d$ particle gives the levels

$$\begin{aligned}[41],\quad &(8,1) \text{ with } l=1,2,3,\cdots,9 \quad (K=1)\\ &(6,2) \text{ with } l=2,3,4,\cdots,8 \quad (K=2)\\ &\qquad\qquad l=0,2,4,6 \qquad\quad (K=0).\end{aligned}$$

The possible partition

$$[5],\quad (10,0) \text{ with } l=0,2,\cdots,10 \quad (K=0)$$

is excluded by the Pauli principle.

If we work in $j-j$ coupling rather than $L-S$ coupling, we have to consider bands with

$$K=\tfrac{5}{2},\tfrac{3}{2},\tfrac{3}{2},\tfrac{1}{2},\tfrac{1}{2},$$

and have to exclude a band with

$$K=\tfrac{1}{2}.$$

In the course of the diagonalization of H we will see, that it is necessary to introduce a band quantum number K, which is identical with the collective model K. Thus, a straightforward procedure would be to drop one of the bands with $K=\frac{1}{2}$ from the further calculation. Another method would be to choose the parameters of the interaction in a way to throw one band with $K=0$, $l_p=2$ in $L-S$ coupling high up and so allow a small admixture in the final lower states. This procedure seems reasonable as the SU_3 wave functions for Ne^{21} are thorough mixtures of 5 particle wave functions. It can be achieved by taking the next contributing term of the interaction (1.2) with $k=4$ into consideration.

† Work submitted as a partial fulfillment of the requirements for a Ph.D. in Physics.
[1] A. M. Lane, Proc. Phys. Soc. (London) **A68**, 197 (1956).
[2] A. E. Litherland, J. A. Kuehner, H. E. Gove, M. A. Clark, and E. Almqvist, Phys. Rev. Letters **7**, 98 (1961).
[3] A. de Shalit, Phys. Rev. **122**, 1530 (1961).
[4] J. P. Elliott, Proc. Roy. Soc. (London) **A245**, 128 (1958).

The Ne^{20} ground-state band is not ideally rotational. We will approximate this situation by taking a variable moment of inertia for the states with different L and retaining simple collective model wave functions for the core. With this approximation the $\frac{7}{2}$ and $\frac{9}{2}$ states turn out too low by approximately 10% and 20%, respectively. If we change the parameters of (1.2) for the $L_c=6$ state by a factor $(1-\alpha)^{1/2}$, it is found that for a value of $\alpha=0.2$ (corresponding to a change of approximately 10% of the wave function), agreement with experiment is improved.

The second section gives a collection of the available experimental data for Ne^{20} and Ne^{21}. It is followed by a review of the previous theoretical interpretations of these data in Sec. 3. In Secs. 4 and 5 a discussion of the Hamiltonian (1.1) and of the diagonalization process is given. In Sec. 6 some moments and transition probabilities are calculated.

2. EXPERIMENTAL DATA

(a) Ne^{20}

The Canadian Chalk River Group has carried out quite extensive work on Ne^{20} using the reaction $C^{12}(C^{12},\alpha\gamma)Ne^{20}$. The resulting energy scheme is given in Fig. 1. A slight deviation from the rotational pattern is noticeable. If we assume a form of the spectrum of

$$E=C_L L(L+1)=\frac{\hbar^2}{2\theta_L}L(L+1)\,, \tag{2.1}$$

we find for the ground-state band

$$C_2=0.27\text{ MeV},\quad C_4=0.21\text{ MeV},\quad C_6=0.18\text{ MeV}.$$

The 7.60 MeV level with $L^\pi=6^+$ is not definitely established.

The lifetimes of the 2^+ and 4^+ levels of the ground-state band have been determined by Clark *et al.*[5] by the Doppler shift attenuation method. The results are

$$\begin{aligned}\tau(1.63\text{ MeV})&=(5.6_{-1.2}{}^{+2.8})\times10^{-13}\text{ sec}\\ \tau(4.25\text{ MeV})&=(0.76_{-0.52}{}^{+0.72})\times10^{-13}\text{ sec}.\end{aligned} \tag{2.2}$$

The lifetime for the 1.63 MeV state is in agreement with the value of $(7.6\pm3.3)\times10^{-13}$ sec given by Devons *et al.*[6] and the value of 7.6×10^{-13} sec determined by Lemberg[7] from the Coulomb excitation of Ne^{20}.

(b) Ne^{21}

Experimental investigations of Ne^{21} energy levels have been carried out by several workers, using the reactions $O^{18}(\alpha,n)Ne^{21}$, $Ne^{20}(d,p)Ne^{21}$, $Na^{23}(d,\alpha)Ne^{21}$, and $F^{19}(He^3,p)Ne^{21}$. The results of the papers before 1959 are summarized in Hinds and Middleton,[8] who use the last reaction. A more recent investigation of the reaction $Ne^{20}(d,p)Ne^{21}$ was made by Freeman.[9] Nearly all the 60 levels measured by Hinds and Middleton are obtained in Ref. 9 also. The available spin and parity assignments are mostly determined from the angular distribution curves of the (d,p) reaction and are given by Burrows *et al.*[10]

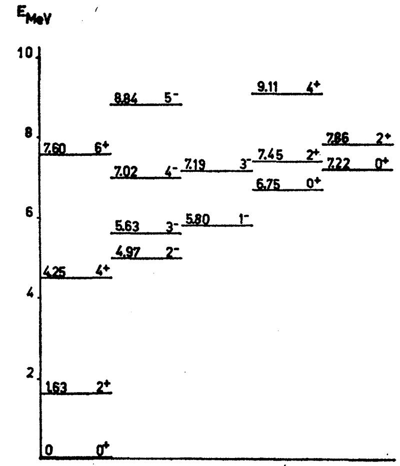

FIG. 1. Energy spectrum of Ne^{20} from Ref. 1.

Using also the *Nuclear Data Sheets*[11], we can infer the illustrated spectrum for Ne^{21} states below 6 MeV. (See Fig. 2.) We can get more information about probable spins and parities of yet unassigned states by considering the mirror nucleus Na^{21}. There is a quite recent summary of the available results by Ajzenberg *et al.*,[12] who investigated the reaction $Ne^{20}(d,n)Na^{21}$.

Analysis of the angular distribution curves was carried out by a distorted-wave Born approximation (DWBA). The 1.73-MeV state gives no agreement for $l=0$, 1, 2, so an assignment of $\frac{7}{2}^+$ is tentatively suggested. The indicated transitions[13] for the 2.86-MeV state would give a J^π of $\frac{7}{2}^+\geq J^\pi\geq\frac{3}{2}^+$ for E_2 and $J^\pi=\frac{5}{2}^+$ for $M1$ transitions, if we assume the 1.73 and 3.57 MeV states to be $\frac{7}{2}^+$ and $\frac{3}{2}^+$, respectively. The $\frac{5}{2}^+$ assignment is supported by the existence of a similar state in that energy region in Na^{23} (see Gove[14]).

The quadrupole moment of the ground state of Ne^{21}

[5] M. A. Clark, H. E. Gove, and A. E. Litherland, Can. J. Phys. **39**, 1241 (1961).
[6] S. Devons, G. Manning, and J. H. Towle, Proc. Phys. Soc. (London) **A69**, 173 (1955).
[7] I. Kh. Lemberg, in *Proceedings of the Second Conference on Reactions between Complex Nuclei, 1960*, edited by A. Zucker, E. C. Halbert, and F. T. Howard (John Wiley & Sons, Inc., New York, 1960), p. 118.

[8] S. Hinds and R. Middleton, Proc. Phys. Soc. (London) **74**, 779 (1959).
[9] J. Freeman, Phys. Rev. **120**, 1436 (1960).
[10] H. B. Burrows, T. S. Green, S. Hinds, and R. Middleton, Proc. Phys. Soc. (London) **A69**, 310 (1956).
[11] *Nuclear Data Sheets*, compiled by K. Way *et al.* (Printing and Publishing Office, National Academy of Sciences–National Research Council, Washington 25, D. C.), NRC60-5-3.
[12] F. Ajzenberg-Selove, C. Cranberg, and F. S. Dietrich, Phys. Rev. **124**, 1548 (1961).
[13] P. M. Endt, and C. van der Leun, Nucl. Phys. **34**, 11 (1962).
[14] H. E. Gove, in *Proceedings of the International Conference on Nuclear Structure, Kingston*, edited by D. A. Bromley and E. W. Vogt (University of Toronto Press, Toronto, 1960), p. 450.

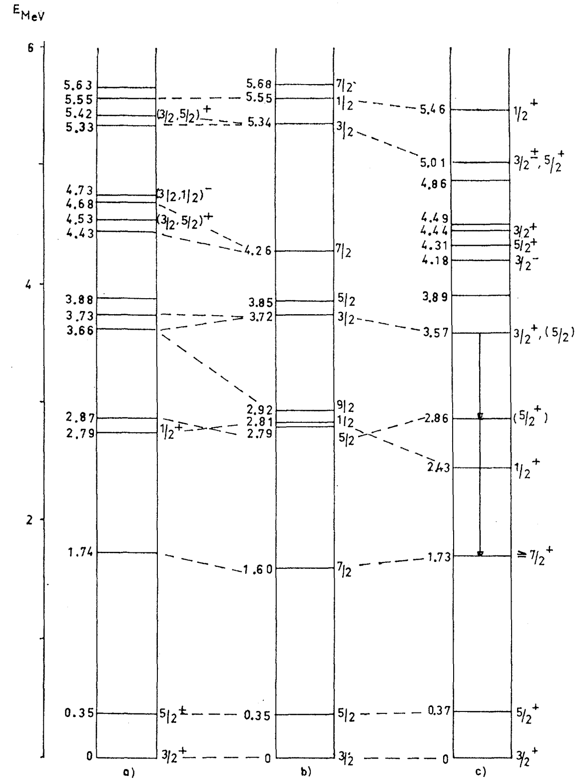

FIG. 2. Comparison of measured and calculated energy spectrum of Ne^{21}. (a) Energy spectrum of Ne^{21} from Refs. 8–9 and *Nuclear Data Sheets* (See Ref. 11). (b) Calculated energy spectrum of Ne^{21} case (b). (c) Energy spectrum of Na^{21} from Refs. 12, 13. (The Na^{21} ground state is 3.53 MeV above the Ne^{21} ground state.)

has been determined by Grosoff *et al.*[15] from the hyperfine structure as

$$\mathrm{QM}=e(+0.093\pm0.010)\times10^{-24}\ \mathrm{cm}^2. \tag{2.3}$$

The magnetic moment of this state is given by La Tourette *et al.*[16] as

$$\mathrm{MM}=-0.662\ \mathrm{nm}, \tag{2.4}$$

using the molecular beam magnetic resonance technique and taking into account the diamagnetic corrections.

Andrev *et al.*[17] have investigated the Coulomb excitation of the first excited level (the same results are reported also by Lemberg[18]). They found that the transition $\frac{3}{2}^+\rightarrow\frac{5}{2}^+$ was mainly of the $E2$ type with a reduced matrix element of

$$B(E2,\tfrac{3}{2}^+\rightarrow\tfrac{5}{2}^+)=0.025\times10^{-48}\ \mathrm{cm}^4. \tag{2.5}$$

This value would correspond to a partial lifetime for the $\frac{5}{2}^+$ state of

$$\tau_{E2}\approx9.2\times10^{-10}\ \mathrm{sec}. \tag{2.6}$$

A direct measurement of the lifetime by Khabakhpashev and Tsenter[19] gives

$$\tau=(6.2\pm6.2)\times10^{-11}\ \mathrm{sec}, \tag{2.7}$$

and from the angular correlation they determined the transition to be mainly $M1$.[20] Deuchars and Dandy[21] determined the ratio δ of the amplitude of the electric quadrupole transition to the amplitude of the magnetic dipole transition from the angular distribution curves as

$$4\times10^{-3}\leq\delta\leq3\times10^{-2}. \tag{2.8}$$

From the lifetimes given in (2.6) and (2.7) we can calculate the partial lifetime τ_{M1} of the $\frac{5}{2}^+$ state by

$$\tau_{M1}=\frac{\tau_{E2}\tau}{\tau_{E2}-\tau}$$

as

$$\tau_{M1}=(6.67_{-5.78}^{\ +7.61})\times10^{-11}\ \mathrm{sec}. \tag{2.9}$$

With the values (2.6) and (2.9) we find for

$$\delta=\left[\frac{T(E2)}{T(M1)}\right]^{1/2}=\left[\frac{\tau_{M1}}{\tau_{E2}}\right]^{1/2},$$

$$\delta=(27_{-17.2}^{+11})\times10^{-2}.$$

These values of δ are higher than the values given by Deuchars and Dandy.

If we use the estimate

$$B_{\mathrm{sp}}(E2)=\frac{5}{4\pi}(\tfrac{3}{5}R_0^2)^2 \tag{2.10}$$

with

$$R_0=1.2A^{1/3}\times10^{-13}\ \mathrm{cm},$$

for the single-particle transition probability (see for example Alder *et al.*[22]), we find for the ratio F of the measured transition probability (2.5) to the single particle estimate

$$F(\tfrac{3}{2}\rightarrow\tfrac{5}{2})\approx15.$$

The β^+ decay of Na^{21} to the Ne^{21} ground state is superallowed with a log ft of 3.6. There is a branch of 2.2% to the first excited $\frac{5}{2}^+$ state with a log ft of 5.0.[13]

3. PREVIOUS THEORETICAL INTERPRETATIONS

(a) Ne^{20}

A straightforward theoretical interpretation can be given with the collective model assuming a Hamiltonian

$$H_{\mathrm{core}}=C(L)^2 \tag{3.1}$$

[15] G. Grosof, M. Buck, W. Lichten, and I. Rabi, Phys. Rev. Letters **1**, 214 (1958).
[16] J. T. La Tourrette, W. E. Quinn, and N. F. Ramsey, Phys. Rev. **107**, 1202 (1957).
[17] D. S. Andrev, K. I. Erokhin, and I. Kh. Lemberg, Isv. Akad. Nauk SSSR, Ser. Fiz. **24**, 1478 (1960).
[18] I. Kh. Lemberg, in *Proceedings of the Second Conference on Reactions between Complex Nuclei, 1960*, edited by A. Zucker, E. C. Halbert, and F. T. Howard, (John Wiley & Sons, Inc., New York, 1960), p. 126.
[19] A. G. Khabakhpashev, and E. M. Tsenter, Zh. Eksperim. i Teor. Fiz. **37**, 991 (1959) [translation: Soviet Phys.—JETP **10**, 705 (1960)].
[20] A. G. Khabakhpashev, and E. M. Tsenter, Isv. Akad. Nauk SSSR, Ser. Fiz. **23**, 883 (1957).
[21] W. M. Deuchars, and D. Dandy, Proc. Phys. Soc. (London) **77**, 1197 (1961).
[22] K. Alder, A. Bohr, T. Huus, B. Mottelson, and A. Winther, Rev. Mod. Phys. **28**, 432 and 439 (1956).

with eigenfunctions

$$|LM,K_L\rangle = \chi_{\text{intrinsic}}{}^{K_L} Y_{LM}(\vartheta_c,\varphi_c). \tag{3.2}$$

For the ground-state band with $K_L=0$ the energy is then given as in (2.1).

The deviation from the pure rotational character cannot be explained in terms of the usual first-order corrections to the collective rotational bands. The rotation vibration interaction[23]

$$H_{\mathrm{RV}} = -\mathrm{const}L^2(L+1)^2 \tag{3.3}$$

does not account for the deviations (it should be significant at the beginning of a shell, where the rotational spacing is large). The rotation particle coupling (RPC)[24] does not give any contribution, for there is no nearby interacting band with $K_L=1$ and positive parity.

Shell-model type calculations using the SU_3 classification scheme have been carried out by Chacon and Moshinsky[25] and Banerjee *et al.*[26]

(b) Ne^{21}

The simplest version of the shell model predicts a ground state of $\frac{5}{2}^+$ for Ne^{21}, a magnetic moment of -1.91 nm and a quadrupole moment of zero. Flowers[27] has calculated an improved value of -1.27 nm for the ground-state magnetic moment by considering the configuration $(d_{5/2})^5$ with total $J=\frac{3}{2}$ and $T=\frac{1}{2}$. He also indicated that the quadrupole moment of odd neutron nuclei can be improved by an appropriate coupling of proton pairs and neutrons. Rakavy[28] performed some preliminary calculations in the region beyond O^{16} using the Nilsson model. For Na^{23} and similarly for Ne^{21} and Na^{21} he predicts above the ground state of $\frac{3}{2}^+$ and the first excited state of $\frac{5}{2}^+$ a state of $J=\frac{7}{2}^+$. Approximately 1 MeV above the $\frac{7}{2}^+$ state should be a $\frac{1}{2}^+$ level. A $\frac{3}{2}^+$ and a $\frac{5}{2}^+$ level should appear between 3 and 4 MeV. Paul and Montague[29] considered another type of collective model calculation for Na^{23}. They arrange three rotational bands based on $K=\frac{1}{2}, \frac{3}{2}, \frac{5}{2}$ so as to reproduce the lowest $\frac{3}{2}^+$, $\frac{5}{2}^+$, and $\frac{1}{2}^+$ levels by means of the RPC interaction between the bands.

This version was applied to Ne^{21} by Freeman.[30] She obtains roughly the following spectrum for Ne^{21}:

J^π	$\frac{3}{2}^+$	$\frac{5}{2}^+$	$\frac{7}{2}^+$	$\frac{1}{2}^+$	$\frac{5}{2}^+$	$\frac{9}{2}^+$	$\frac{3}{2}^+$	$\frac{5}{2}^+$	$\frac{7}{2}^+$
E_{MeV}	0	0.35	1.35	2.80	2.85	3.00	3.55	5.15	5.25.

A more recent investigation of the Nilsson model in the $s-d$ shell is given by Bhatt.[31] In the case of Ne^{21} he found that for a value of the deformation parameter η of $\eta=3$ a magnetic moment of -0.60 nm can be obtained, while the fit of the energy spectrum is rather poor. The $E2$ transition probability for the $\frac{3}{2}^+$ ground state and $\frac{5}{2}^+$ first excited state transition turns out too low even for $\eta=4$. Furthermore, in the case of larger deformations the agreement with the magnetic moment deteriorates.[30a]

4. THE HAMILTONIAN OF THE SYSTEM $Ne^{21}=Ne^{20}+2s-1d$ NEUTRON

For the H_p-part of the Hamiltonian (1.1) we take

$$H_p = T + V(r_p) - D(\mathbf{s}\cdot\mathbf{l}), \tag{4.1}$$

where $V(r_p)$ is any shell-model single particle central potential and the parameter of the spin-orbit force D is greater than zero.

The corresponding normalized $j-j$ coupling wave function then takes the form

$$|jm_j\rangle = |n=2; jm_j, \tfrac{1}{2}l\rangle = \sum_{m_l,m_s} (\tfrac{1}{2}lj; m_s m_l m_j)|\tfrac{1}{2}m_s\rangle|n=2; lm_l\rangle, \tag{4.2}$$

where $(\frac{1}{2}lj; m_s m_l m_j)$ is a Clebsch-Gordan coefficient and

$$|n=2; lm_l\rangle = R_l^{(2)}(r_p) Y_{lm_l}(\vartheta_p,\varphi_p) \tag{4.3}$$

is an eigenfunction of $H_p' = T + V(r_p)$.

For H_{core} we use the collective model Hamiltonian introduced in (3.1) and (3.2).

The following argument leads to the first term of H_{coupling}: In an optical-model calculation (the shell model is essentially a version of the optical model without the absorptive (imaginary) part of the potential) the spin-orbit force is introduced in the following way.[32] Consider the scattering of a nucleon with spin $\mathbf{s}$ and initial and final momenta $\mathbf{k}_i$, $\mathbf{k}_f$. The lowest order scalar term that can be constructed from these quantities is

$$I_1 \propto \mathbf{s}\cdot(\mathbf{k}_i \times \mathbf{k}_f). \tag{4.4}$$

The spin orbit potential is then given by

$$U_1(r) \propto \int \exp(i\Delta\mathbf{k}\cdot\mathbf{r})[\mathbf{s}\cdot(\mathbf{k}_i\times\mathbf{k}_f)]\rho(\Delta\mathbf{k})d^3(\Delta k), \tag{4.5}$$

where $\Delta\mathbf{k} = \mathbf{k}_f - \mathbf{k}_i$ is the momentum transfer, and ρ is

[23] S. A. Moszkowski, in *Handbuch der Physik*, edited by S. Flügge (Springer-Verlag, Berlin, 1957), Vol. 39, p. 532.
[24] A. K. Kerman, Kgl. Danske Videnskab. Selskab, Mat. Fys. Medd. 30, 4 (1956).
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[28] G. Rakavy, Nucl. Phys. 4, 375 (1957).
[29] E. B. Paul, and J. H. Montague, Nucl. Phys. 8, 61 (1958).
[30] J. Freeman, in *Proceedings of the International Conference on Nuclear Structure, 1960*, edited by D. A. Bromley and E. Vogt (University of Toronto Press, Toronto, 1960), p. 447.
[31] K. H. Bhatt, Nucl. Phys. 39, 375 (1962).
[30a] *Note added in proof.* While this article was in press a survey of the odd A nuclei in the s-d shell using the asymmetric core collective model was published by Chi and Davidson [Phys. Rev. 131, 366 (1963)]. Their more detailed model employs a smaller number of parameters than the usual Nilsson model and the model presented in this paper. While the results show a good fit of the energy spectra and a reasonable fit of the ground state magnetic moments, the ground state quadrupole moments and the lifetimes of the first excited states do not agree so well with the measured values.
[32] D. C. Peaslee, Ann. Rev. Nucl. Sci. 5, 118 (1955).

the density distribution of the scatterer (presumed to be spherically symmetric). Evaluation of the Fourier integral (4.5) gives

$$U_1(r) \propto \frac{1}{r}\frac{\partial\rho(r)}{\partial r}(\mathbf{s}\cdot\mathbf{l}), \tag{4.6}$$

where $\mathbf{l}$ is the angular momentum of the particle.

In our case we are not only concerned with the scattered particle, but with the interacting particle (or particle group) in the scatterer too. So we have to work in the corresponding center-of-mass system and consider the term

$$I_2 \propto \mathbf{s}\cdot(\mathbf{K}_i\times\mathbf{K}_f), \tag{4.7}$$

where $\mathbf{K}_i$, $\mathbf{K}_f$ are the initial and final momenta of the struck particle.

If we take into account that in the c.m. system the momenta of the scattered and the struck particle are in opposite directions, evaluation of the corresponding Fourier integral yields

$$U_2(r) \propto -\frac{1}{r}\frac{\partial\rho'(r)}{\partial r}(\mathbf{s}\cdot\mathbf{L}), \tag{4.8}$$

where $\mathbf{L}$ is the angular momentum of the struck particle.

If we take the proportionality constants in (4.6), (4.8) (including the Thomas term) negative, we get besides the spin-orbit force in Eq. (4.1) the term

$$+D'(\mathbf{s}\cdot\mathbf{L}), \tag{4.9}$$

which couples the particle spin to the angular momentum of the core.

As ρ represents the density of the whole core, ρ' the density of the core minus the struck particle group, we should have to the first order $D'\lesssim D$, but exchange terms are neglected in this argument. If we believe in this phenomenological approach, we can say the following. In other rotational nuclei where a particle with a $J=\frac{1}{2}^+$ could be coupled to a rotational core (see Ref. 22,), we find that the $\frac{3}{2}^+$ and $\frac{5}{2}^+$ levels, which would be degenerate in the absence of spin-orbit forces, split up into a lower $\frac{3}{2}$ and a higher $\frac{5}{2}$ state. This could be interpreted by a force of the type (4.9). (The collective model uses the "decoupling" term.)

For the coupling of the particle [described by Eqs. (4.1) and (4.2)] and the core we use a 2^k-2^k pole force of the usual form

$$H_{c,p}=\textstyle\sum_k \alpha_k(R_c,r_p)(2k+1)P^k(\cos\theta_{c,p}). \tag{4.10}$$

For the coupling of a particle with $j=\frac{1}{2},\frac{3}{2},\frac{5}{2}$ to a core with an angular momentum of $L=0, 2, 4, \cdots$ only the terms with $k=2, 4$ give contributions, so that

$$H_{c,p}=H_2+H_4, \tag{4.11}$$

where

$$H_2=\alpha_2(R_c,r_p)\sum_{M_i=-2}^{+2}(-)^{M_i}\times Y_{2,M_i}(\vartheta_c,\varphi_c)Y_{2,-M_i}(\vartheta_p,\varphi_p), \tag{4.12}$$

$$H_4=\alpha_4(R_c,r_p)\sum_{M_i=-4}^{+4}(-)^{M_i}\times Y_{4,M_i}(\vartheta_c,\varphi_c)Y_{4,-M_i}(\vartheta_p,\varphi_p).$$

The total Hamiltonian of Ne^{21} then takes the form

$$H=H_c+H_p+D(\mathbf{s}\cdot\mathbf{L})+H_2+H_4, \tag{4.13}$$

where the different parts are given by (3.1), (4.1) and (4.12).

5. DIAGONALIZATION OF THE HAMILTONIAN H

The diagonalization of H is carried out in the following steps. We choose as a zero-order wave function

$$|JM,Lj\rangle=\sum_{M_L,m_j}(LjJ;M_Lm_jM)|LM_L\rangle|jm_j\rangle, \tag{5.1}$$

the Clebsch-Gordan coupled wave function of the core and the outside particle, and consider the part

$$\begin{aligned}H(1)&=H_p'+H_2\\&=T+V(r_p)+H_2,\end{aligned} \tag{5.2}$$

of the Hamiltonian.

In the representation (5.1) the matrix elements of H_p' are already diagonal and we fix our preliminary energy scale by taking

$$\langle J,Lj|H_p'|J,Lj\rangle=E\delta_{j,\frac{1}{2}}, \tag{5.3}$$

where E is the separation of the $l=0$ and $l=2$ levels. We can omit the dependence on the magnetic quantum numbers, as no term in our Hamiltonian splits this degeneracy.

The diagonalization of H_2 can be carried out explicitly, if we retain j as a good quantum number. Introducing

$$Q_2=+(f_2/\pi)\langle l=2|\alpha_2(r_p)|l=2\rangle\times\langle\text{intr}|\alpha_2(R_c)|\text{intr}\rangle, \tag{5.4}$$

under the assumption that $\alpha_2(R_c,r_p)$ is of the form $f_2\alpha_2(r_p)\alpha_2(R_c)$ and a new quantum number K, which will be discussed below, we obtain the following diagonal elements (independent of J):

$$\begin{aligned}
&j=\tfrac{5}{2}\\
&\quad\langle J,\tfrac{5}{2}K_1|H_2|J,\tfrac{5}{2}K_1\rangle=-(5/14)Q_2\\
&\quad\langle J,\tfrac{5}{2}K_2|H_2|J,\tfrac{5}{2}K_2\rangle=(1/14)Q_2\\
&\quad\langle J,\tfrac{5}{2}K_3|H_2|J,\tfrac{5}{2}K_3\rangle=(2/7)Q_2\\
&j=\tfrac{3}{2}\\
&\quad\langle J,\tfrac{3}{2}K_2'|H_2|J,\tfrac{3}{2}K_2'\rangle=-\tfrac{1}{4}Q_2\\
&\quad\langle J,\tfrac{3}{2}K_3'|H_2|J,\tfrac{3}{2}K_3'\rangle=\tfrac{1}{4}Q_2\\
&j=\tfrac{1}{2}\\
&\quad\langle J,\tfrac{1}{2}K_3''|H_2|J,\tfrac{1}{2}K_3''\rangle=0.
\end{aligned} \tag{5.5}$$

The corresponding eigenfunctions are of the form

$$|JM,jK\rangle=\sum_L a_K{}^{J,j}(L)|JM,Lj\rangle, \qquad (5.6)$$

where the expansion coefficients $a_K{}^{J,j}(L)$ can be calculated with standard methods.

It can be seen in the following way, that the quantum number K introduced here is identical with the collective model K_j, the projection of the particle angular momentum on a body-fixed axis. We rewrite Eq. (5.6) by transforming the right-hand side to a nuclear coordinate system as

$$|JM,jK\rangle=\sum_{K_j}\sum_L a_K{}^{J,j}(L)\left[\frac{2(2L+1)}{(2J+1)}\right]^{1/2} \times(LjJ;0K_jK_j)\Psi_{K_jM}{}^J(L), \qquad (5.7)$$

where

$$\Psi_{K_jM}{}^J(L)=\left[\frac{2J+1}{16\pi^2}\right]^{1/2}\chi_{K_L=0}{}^{(L)}\{\mathfrak{D}_{MK_j}{}^J|jK_j\rangle +(-)^{J-j}\mathfrak{D}_{M-K_j}{}^J|j-K_j\rangle\}, \qquad (5.8)$$

is a strong coupling wave function, if the intrinsic core function $\chi_0{}^{(L)}$ in the body-fixed system is independent of L (purely rotational core). It should be stressed at this point that we use a spherical rotator for the core ($K_L=0$) rather than the collective model picture (see Discussion C). The $\mathfrak{D}$ are symmetric top eigenfunctions and $|jK_j\rangle$ are the odd-neutron eigenfunctions in the body-fixed system. If we have a purely rotational core, we can execute the sum over L. For a proper choice of phase of the $a_K{}^{J,j}(L)$ we obtain

$$\sum_L a_K{}^{J,j}(L)\left[\frac{2(2L+1)}{(2J+1)}\right]^{1/2}(LjJ;0K_jK_j)=\delta_{KK_j}. \qquad (5.9)$$

So we see that in this case our eigenfunctions (5.6) are identical with the strong coupling wave functions (5.8). We then have

$$K_1=\tfrac{5}{2}$$
$$K_2=K_2'=\tfrac{3}{2} \qquad (5.10)$$
$$K_3=K_3'=K_3''=\tfrac{1}{2},$$

and the $a_K{}^{J,j}(L)$ can be given by the closed expression

$$a_K{}^{J,j}(L)=\left[\frac{2(2L+1)}{(2J+1)}\right]^{1/2}(LjJ;0KK). \qquad (5.11)$$

The terms H_2 and H_4 are invariant under rotations. Then we can choose a coordinate system (see e.g. Rose[33]), that

$$H_k\propto Y_{k0}(\theta,0).$$

As the coupling rule for spherical harmonics gives

$$Y_{40}=b_1(Y_{20})^2+b_2Y_{00}+b_3Y_{20},$$

we find that the eigenfunctions (5.6) diagonalize H_4 also. We obtain

$$\langle J,\tfrac{1}{2}K|H_4|J,\tfrac{1}{2}K\rangle=0$$
$$\langle J,\tfrac{3}{2}K|H_4|J,\tfrac{3}{2}K\rangle=0$$
$$\langle J,\tfrac{5}{2}\tfrac{5}{2}|H_4|J,\tfrac{5}{2}\tfrac{5}{2}\rangle=(3/28)Q_4 \qquad (5.12)$$
$$\langle J,\tfrac{5}{2}\tfrac{3}{2}|H_4|J,\tfrac{5}{2}\tfrac{3}{2}\rangle=-(9/28)Q_4$$
$$\langle J,\tfrac{5}{2}\tfrac{1}{2}|H_4|J,\tfrac{5}{2}\tfrac{1}{2}\rangle=(6/28)Q_4,$$

with

$$Q_4=\frac{f_4}{\pi}\langle l=2|\alpha_4(r_p)|l=2\rangle\langle\text{intr}|\alpha_4(R_c)|\text{intr}\rangle. \qquad (5.13)$$

If we write

$$(\mathbf{L})^2=(\mathbf{J}-\mathbf{j})^2=\mathbf{J}^2+\mathbf{j}^2-2\mathbf{J}\cdot\mathbf{j} \qquad (5.14)$$

we obtain the rotational part of the collective model Hamiltonian for an odd-A nucleus.[34] The coupling term

$$-2(\mathbf{J}\cdot\mathbf{j}-\mathbf{J}_3\cdot\mathbf{j}_3) \qquad (5.15)$$

represents the RPC.[24] The diagonal elements of H_c in the representation (5.6) can be written in the form

$$\langle J,jK|H_c|J,jK\rangle=C\{J(J+1)+j(j+1) -2K^2+(-)^{J+j}(j+\tfrac{1}{2})(J+\tfrac{1}{2})\delta_{K\frac{1}{2}}\}, \qquad (5.16)$$

which corresponds to the expression used in the collective model, if we take a value of

$$(-)^{j-\frac{1}{2}}(j+\tfrac{1}{2})$$

for the decoupling parameter (Ref. 34). This choice stems from the special form of the strong coupling wave function (5.8).

For the final diagonalization of the total Hamiltonian H in the representation (5.6), we have to consider the following parameters:

$$Q_2\ Q_2'\ Q_4;D;D_2'D_0';E;C. \qquad (5.17)$$

Here

$$Q_2'=(f_2/\pi)\langle l=2|\alpha_2(r_p)|l=0\rangle\langle\text{intr}|\alpha_2(R_c)|\text{intr}\rangle \qquad (5.18)$$

is the H_2 interaction parameter between $j=\frac{1}{2}$ and $j=\frac{3}{2}$, $\frac{5}{2}$ states. The parameter D' for the coupling of particle spin and angular momentum of the core is subscripted to allow for a different coupling strength in the case of a $j=\frac{1}{2}$ particle (D_0') and a $j=\frac{3}{2}$, $\frac{5}{2}$ particle (D_2').

If we assume the term H_2+H_4 to stem from a short-range force of the Yukawa type

$$V(\mathbf{r}_p,\mathbf{R}_c)\propto\frac{\exp(-\mu|\mathbf{R}_c-\mathbf{r}_p|)}{\mu|\mathbf{R}_c-\mathbf{r}_p|}, \qquad (5.19)$$

we find after suitable expansion the relations

$$+0.33\lesssim(Q_4/Q_2)\lesssim+1.00$$
$$-0.60\lesssim(Q_2'/Q_2)\lesssim-0.10 \quad \text{for} \quad 0\leq\mu\leq\infty, \qquad (5.20)$$

by using harmonic oscillator wave functions for the

[33] M. E. Rose, *Elementary Theory of Angular Momentum* (John Wiley & Sons, Inc., New York, 1961), p. 94.

[34] S. A. Moszkowski, in *Handbuch der Physik*, edited by S. Flügge (Springer-Verlag, Berlin, 1957), Vol. 39, p. 482, 487.

particle and the radial density distribution

$$\rho(R_c)=\sum_{\Lambda=0}^{\Lambda=1(2)}\sum_{l}2(2l+1)u_{\Lambda l}^2(R_c)$$

(with radial harmonic oscillator $u_{\Lambda l}$) for the core.

If we assume H_2+H_4 to be a long-range potential of the usual 2^k pole interactions with

$$\alpha_2=f_2(\mu' r_p)^2(\mu' R_c)^2 \qquad (5.21)$$
$$\alpha_4=f_4(\mu' r_p)^4(\mu' R_c)^4,$$

we get with the same wave functions for core and particle as used in the calculation of (5.20)

$$(Q_4/Q_2)=15(f_4/f_2)(\mu'/a)^4 \qquad (5.22)$$
$$(Q_2'/Q_2)=-0.9,$$

where a is the inverse of the "shape parameter" for the harmonic oscillator wave functions.

For the parameter D we can get an estimate from O^{17} (see Ajzenberg-Selove and Lauritsen[35]): $D=2$ MeV. Kurath[36] has examined the variation of this parameter in the p shell. He found that the curve of D plotted against the total number of nucleons A gives a smooth increase up to $A=8$ with $D\approx 2$ MeV, a steep increase between $A=9$–12 and a smooth increase to $D\approx 5$ MeV at the end of the shell. If we anticipate a similar behavior in the $s-d$ shell, we would have a parameter D between 2.5 and 4.5 MeV. D_0' and D_2' are essentially free parameters, though one would expect

$$D\geq D_0'\approx D_2'\geq 0.$$

To account for the differing C values of the L states of the core, we calculate the matrix elements of H_c with the energy values of the ground-state band of Ne^{20} as given in Fig. 1. The separation of the $l=2$ particle and the $l=0$ particle E can be taken from O^{17} to be approximately -1.1 MeV. With the filling of the shell one would expect this separation to get smaller, as the centrifugal repulsion on the d particles decreases.

As pointed out in the Introduction there are two ways to account for antisymmetrization: (a) Not taking the $j=\frac{5}{2}$, $K=\frac{1}{2}$ band into account, assuming that the $j=\frac{5}{2}$ subshell is filled first. (b) Lifting a band with $K=0$, $l=2$ in $L-S$ coupling up by a proper choice of the parameters. The final diagonalization of the Hamiltonian H for these two possibilities was carried out on an IBM 1620 with various sets of the parameters (5.17). It was found that the parameters (in units of MeV):

$$\text{(a)}\quad Q_2=-6.25 \quad Q_2'=+4.00 \quad Q_4=0$$
$$D=4 \quad D_0'=0.5 \quad D_2'=0.4 \qquad (5.23\text{a})$$
$$E=-0.15$$

$$\text{(b)}\quad Q_2=+9.80 \quad Q_2'=-7.70 \quad Q_4=15.85$$
$$D=3.3 \quad D_0'=1.2 \quad D_2'=0.7 \qquad (5.23\text{b})$$
$$E=-0.71$$

(and

$$C_2=0.27 \quad C_4=0.21 \quad C_6=0.18$$

in both cases) yield a fair fit for the $\frac{1}{2}^+$, $\frac{3}{2}^+$, $\frac{5}{2}^+$, and $\frac{7}{2}^+$ states of Ne^{21} below 6 MeV.

The spin-core coupling strength for the $l=0$ particle is slightly bigger than for the $l=2$ particle in both cases. This might be due to the fact that in the $l=0$ case the spin-orbit strength is zero and for this reason the spin couples more strongly to the core. The parameters D and E show a greater deviation from the O^{17} values in case (a) than in case (b). The values of Q_2, Q_2', and Q_4 favor the long-range case, though in both cases we find a smaller ratio of $|Q_2'/Q_2|$ than the estimate of 0.9 obtained with harmonic oscillator wave functions.

If we readjust our energy scale by putting $E_{\text{ground}}=0$, we can list the following levels:

(a) $J=\frac{1}{2}^+$: 2.77, 10.01 (MeV)
$J=\frac{3}{2}^+$: 0 , 3.67, 8.76, 12.09 (MeV)
$J=\frac{5}{2}^+$: 0.37, 2.83, 4.81, 11.62, 13.88 (MeV)
$J=\frac{7}{2}^+$: 1.52, 4.37, 5.83, 10.10, 14.37 (MeV)
$J=\frac{9}{2}^+$: 2.81, 6.00, 7.84, 14.32, 17.55 (MeV)
(5.24a)

(b) $J=\frac{1}{2}^+$: 2.81, 5.55, 24.25 (MeV)
$J=\frac{3}{2}^+$: 0 , 3.72, 5.34, 10.38, 23.70 (MeV)
$J=\frac{5}{2}^+$: 0.35, 2.79, 3.85, 8.17, 11.65 26.58 (MeV)
$J=\frac{7}{2}^+$: 1.60, 4.26, 5.68, 6.94, 13.16, 25.31 (MeV)
$J=\frac{9}{2}^+$: 2.92, 5.68, 6.49, 11.88, 14.81, 30.09 (MeV)
(5.24b)

(The states above 20 MeV in case (b) are nearly pure $K=0$, $l=2$ states, as can be seen by transforming representation (5.6) into the $L-S$ coupling picture.) The

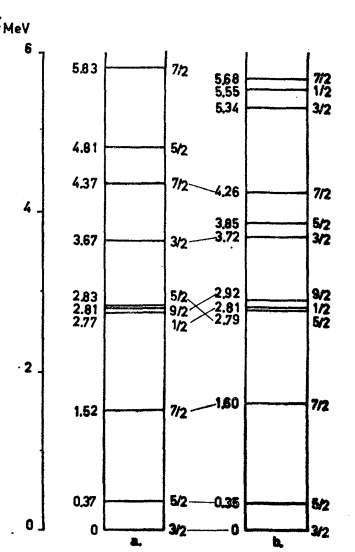

FIG. 3. Calculated energy spectrum of Ne^{21} for cases (a) and (b).

[35] F. Ajzenberg-Selove and T. Lauritsen, Nucl. Phys. 11, 222 (1959).
[36] D. Kurath, Phys. Rev. 101, 216 (1956).

TABLE I. Comparison of theoretical and experimental energy levels of Ne^{21} below 6 MeV.

Calc. (a)			Calc. (b)			Meas.		
E	J	Dev.	E	J	Dev.	E	J	Comments on assignment of meas. level
0	$\frac{3}{2}$	···	0	$\frac{3}{2}$	···	0	$\frac{3}{2}$	
0.37	$\frac{5}{2}$	+6%	0.35	$\frac{5}{2}$	···	0.35	$\frac{5}{2}$	
1.52	$\frac{7}{2}$	−12%	1.60	$\frac{7}{2}$	−8%	1.74	($\frac{7}{2}$)	Suggested for Na^{21} by Ref. 12.
2.77	$\frac{1}{2}$	−1%	2.81	$\frac{1}{2}$	+2%	2.79	$\frac{1}{2}$	
2.83	$\frac{5}{2}$	−1%	2.79	$\frac{5}{2}$	−3%	2.87	($\frac{5}{2}$)	From transitions in Na^{21}.
2.81	$\frac{9}{2}$	−25%	2.92	$\frac{9}{2}$	−20%	3.66 3.73	?	A $\frac{9}{2}$ level in Na^{23} at 2.71 (see Ref. 13, p. 5) would support a $\frac{9}{2}$ level in that region.
3.67	$\frac{3}{2}$	··· −2%	3.72	$\frac{3}{2}$	+2% ···	3.66 3.73	? ?	From comparison with Na^{21} one of these levels should be a $\frac{3}{2}$.
···	···	···	3.85	$\frac{5}{2}$	−1%	3.88	?	
4.37	$\frac{7}{2}$	−1% −7%	4.26	$\frac{7}{2}$	−4% −10%	4.43 4.68	? ?	No $\frac{7}{2}$ level observed in this region.
4.81	$\frac{5}{2}$	+6%	···	···	···	4.53	($\frac{5}{2}$,$\frac{3}{2}$)	
···	···	···	5.34	$\frac{3}{2}$	···	5.33	?	Could be 5.42 ($\frac{3}{2}$,$\frac{5}{2}$).
···	···	···	5.55	$\frac{1}{2}$	···	5.55	?	$\frac{1}{2}$ level in Na^{21} at 5.47 MeV. 5.33 or 5.55 levels open to the assignment of $\frac{1}{2}$.
···	···	···	5.68	$\frac{7}{2}$,$\frac{9}{2}$	?			
5.83	$\frac{7}{2}$	?						

high-level density and paucity of spin and parity assignments for the experimental levels above 6 MeV do not allow any definite interpretation in this region.

The possible breaking off of the rotational spectrum for Ne^{20} or the fact that we put all the deviation from the pure rotational structure into the parameter C instead of modifying the wave functions, could well account for the growing deviations from the measured levels in the case of the $\frac{7}{2}$ and $\frac{9}{2}$ states.

We write $Q_2(L,L')$, $Q_2'(L,L')$, $Q_4(L,L')$ instead of the parameters Q_2, Q_2', Q_4, which are independent of L and introduce the following modifications:

$$Q(L,L')=Q \text{ for } L,L'\neq 6$$
$$Q(6,L)\ =Q(L,6)=(1-\alpha)^{1/2}Q$$
$$Q(6,6)\ =(1-\alpha)Q.$$

This change effects only states with $J\geq\frac{7}{2}$. If we calculate the matrix of the total Hamiltonian H for $J=\frac{7}{2},\frac{9}{2}$ in representation (5.1) and carry out the diagonalization with the parameters (5.23b) for different values of α, we find that for $\alpha=0.2$ the lowest $\frac{7}{2}$ level coincides with the measured level within 1% and the lowest $\frac{9}{2}$ level shows a deviation of approximately 7% only. (Compare Table I). We have not accounted for some positive parity levels in the region 4–5 MeV. They could probably arise from the coupling of a $f-p$ particle of a p hole to the negative parity states of Ne^{20}, or from the coupling of a $s-d$ particle to the higher positive parity bands or a more complicated configuration. (For level schemes see Figs. 2 and 3.)

The final eigenfunctions take the form

$$|J,M\rangle=\sum_{j,K} c^J(j,K)\,|J,MjK\rangle, \qquad (5.25)$$

where the $c^J(j,K)$ can be given numerically. For further convenience we note the c's of the ground state and the first excited $\frac{5}{2}^+$ and $\frac{1}{2}^+$ states for cases (a) and (b) in Table II.

6. CALCULATION OF MOMENTS AND TRANSITIONS

The quadrupole moment operator of our system can be written as

$$Q_{Ne^{21}}=Q_{core}+Q_n\,,$$

where Q_n is zero. Q_c is given by the usual definition

$$Q_c=QM=Ze\left[\frac{16\pi}{5}\right]^{1/2}R_c^2Y_{20}(\vartheta_c,\varphi_c)\,. \qquad (6.1)$$

If we evaluate the quadrupole moment for the ground state $J=M=\frac{3}{2}$ using the representation (5.25) we ob-

TABLE II. Expansion coefficients $c^J(j,K)$ in the final wave functions.

			$c^J(j,K)$	
J	j	K	(a)	(b)
$\frac{1}{2}$	$\frac{5}{2}$	$\frac{1}{2}$	···	0.7802
	$\frac{3}{2}$	$\frac{1}{2}$	0.1992	0.4632
	$\frac{1}{2}$	$\frac{1}{2}$	0.9800	0.4205
$\frac{3}{2}$	$\frac{5}{2}$	$\frac{3}{2}$	0.9945	0.9947
		$\frac{1}{2}$	···	0.0478
	$\frac{3}{2}$	$\frac{3}{2}$	−0.0920	−0.0849
		$\frac{1}{2}$	−0.0477	−0.0107
	$\frac{1}{2}$	$\frac{1}{2}$	−0.0165	+0.0324
$\frac{5}{2}$	$\frac{5}{2}$	$\frac{5}{2}$	0.3962	0.3584
		$\frac{3}{2}$	0.9117	0.9171
		$\frac{1}{2}$	···	0.1405
	$\frac{3}{2}$	$\frac{3}{2}$	−0.0958	−0.0880
		$\frac{1}{2}$	−0.0491	0.0281
	$\frac{1}{2}$	$\frac{1}{2}$	−0.0147	0.0463

tain for the cases (a) and (b)

$$\langle \tfrac{3}{2}\tfrac{3}{2}|QM|\tfrac{3}{2}\tfrac{3}{2}\rangle = e(0.0025Q_{20}+0.2421Q_{22}+0.1938Q_{42}-0.0404Q_{44}), \quad (6.2a)$$

$$\langle \tfrac{3}{2}\tfrac{3}{2}|QM|\tfrac{3}{2}\tfrac{3}{2}\rangle = e(0.0029Q_{20}+0.2515Q_{22}+0.1743Q_{42}-0.0315Q_{44}), \quad (6.2b)$$

where the $Q_{LL'}=Q_{L'L}$ are the intrinsic moments defined by

$$Q_{LL'}=Z\langle \mathrm{intr}(L)|R_c^2|\mathrm{intr}(L')\rangle. \quad (6.3)$$

If we assume the intrinsic moments all to equal Q_0 (this should be the case for a purely rotational core) we have

$$\langle \tfrac{3}{2}\tfrac{3}{2}|QM|\tfrac{3}{2}\tfrac{3}{2}\rangle = e(0.398)Q_0 \quad (6.4a)$$

$$= e(0.397)Q_0. \quad (6.4b)$$

We can determine the quantities Q_{20} and Q_{42} from the lifetimes of the 2^+ and 4^+ states of the Ne^{20} ground-state band, which have been given in (2.2).

We use the formula (see Elliott & Lane,[37])

$$[\tau(E2;L_i\rightarrow L_f)]^{-1}=T(E2;L_i\rightarrow L_f) = \frac{1}{\hbar}\frac{4\pi}{75}\left(\frac{E_\gamma}{\hbar c}\right)^5 e^2B(E2;L_i\rightarrow L_f), \quad (6.5)$$

where the reduced transition probabilities are given by (using the notation of Rose[33] and Condon-Shortley[38])

$$B(E2;L_i\rightarrow L_f)=|\langle L_i\|ZR_c^2Y_2(c)\|L_f\rangle|^2 = \frac{(2L_f+1)}{(2L_i+1)}|\langle L_f|ZR_c^2Y_2(c)|L_i\rangle|^2. \quad (6.6)$$

The second formula follows from the principle of detailed balance.

As we have

$$\begin{aligned} B(E2;2\rightarrow 0) &= (1/4\pi)(Q_{20})^2 \\ B(E2;4\rightarrow 2) &= (5/14\pi)(Q_{42})^2, \end{aligned} \quad (6.7)$$

we obtain from Eq. (6.5)

$$\begin{aligned} (Q_{20})^2 &= (4\pi/1.41)10^{-62}(1/\tau_{20}) \\ (Q_{42})^2 &= (14\pi/7.56)10^{-63}(1/\tau_{40}). \end{aligned} \quad (6.8)$$

The sign of $Q_{LL'}$ is not determined by experiment; to fit the data we take it positive, which corresponds in the language of the collective model to a prolate shape for Ne^{20}. (Nilsson model $\eta>0$.) Then we find from Eq. (6.8) and the values for the lifetimes

$$\begin{aligned} Q_{20} &= (3.99_{-0.73}^{+0.51})\times 10^{-25}\ \mathrm{cm}^2 \\ Q_{42} &= (2.77_{-0.79}^{+2.15})\times 10^{-25}\ \mathrm{cm}^2. \end{aligned} \quad (6.9)$$

If we insert the mean value

$$Q_0=(3.38_{-0.76}^{+1.33})\times 10^{-25}\ \mathrm{cm}^2$$

of these quantities into Eq. (6.4) we find for the quadrupole moment

$$\langle \tfrac{3}{2}\tfrac{3}{2}|QM|\tfrac{3}{2}\tfrac{3}{2}\rangle = e(0.134_{-0.030}^{+0.053})\times 10^{-24}\ \mathrm{cm}^2, \quad (6.10)$$

for cases (a) and (b). This is higher than the measured value of $e(0.093\pm0.01)\times10^{-24}$ cm^2.

If we want to be more accurate we have to consider (6.2). Since we do not know the values of Q_{22} and Q_{44}, we can only get a linear equation in these quantities

$$(0.38_{-0.41}^{+0.16})\times 10^{-25}=(0.2421Q_{22}-0.0404Q_{44}) \quad (6.11a)$$

$$(0.44_{-0.38}^{+0.14})\times 10^{-25}=(0.2515Q_{22}-0.0315Q_{44}). \quad (6.11b)$$

To determine Q_{22} and Q_{44} we need further experimental information. We can get this form the measured value of the $\tfrac{3}{2}^+\rightarrow\tfrac{5}{2}^+$ $E2$ transition, given in (2.5). If we calculate the $E2$ transition with representation (5.25) according to (6.6) we obtain

$$B(E2;\tfrac{3}{2}\rightarrow\tfrac{5}{2})=|\langle\tfrac{3}{2}\|ZR_c^2Y_2(c)\|\tfrac{5}{2}\rangle|^2 = \frac{1}{14\pi}(1.3250Q_{20}-0.1028Q_{22}+1.1713Q_{42}+0.3560Q_{44})^2 \quad (6.12a)$$

$$B(E2;\tfrac{3}{2}\rightarrow\tfrac{5}{2})=\frac{1}{14\pi}(1.4471Q_{20}-0.0245Q_{22}+1.0625Q_{42}+0.2836Q_{44})^2. \quad (6.12b)$$

If we assume again that the intrinsic moments are equal, we have

$$B(E2;\tfrac{3}{2}\rightarrow\tfrac{5}{2})=(0.020_{-0.008}^{+0.019})\times10^{-48}\ \mathrm{cm}^4 \quad (6.13)$$

for both cases. The measured value of 0.025×10^{-48} falls well within these limits.

If, on the other hand, we take the same procedure as for the quadrupole moment, we get a second linear equation in Q_{22} and Q_{44}

$$(1.96_{-0.72}^{+1.89})\times10^{-25}=-0.1028Q_{22}+0.3560Q_{44}, \quad (6.14a)$$

$$(1.77_{-3.02}^{+1.90})\times10^{-25}=-0.0245Q_{22}+0.2836Q_{44}. \quad (6.14b)$$

Solution of the system (6.11) and (6.14) yields

$$\begin{aligned} Q_{22} &= (2.62_{-3.38}^{+1.61})\times10^{-25}\ \mathrm{cm}^2 \\ Q_{44} &= (6.25_{-9.95}^{+5.78})\times10^{-25}\ \mathrm{cm}^2 \end{aligned} \quad (6.15a)$$

$$\begin{aligned} Q_{22} &= (2.54_{-2.87}^{+1.40})\times10^{-25}\ \mathrm{cm}^2 \\ Q_{44} &= (6.46_{-10.91}^{+6.80})\times10^{-25}\ \mathrm{cm}^2. \end{aligned} \quad (6.15b)$$

We see that the fit of the given experimental data leaves us too wide a scope for the quantities Q_{22} and Q_{44}. For a more accurate determination we would need further experimental data on the remaining $E2$ transitions.

The operator of the magnetic moment for our system is defined by

$$\boldsymbol{\mu}=\mu_0(g_c\mathbf{L}+g_l\mathbf{l}+g_s\mathbf{s}), \quad (6.16)$$

[37] J. P. Elliott, and A. M. Lane, in *Handbuch der Physik*, edited by S. Flügge, (Springer-Verlag, Berlin, 1957), Vol. 39, p. 256.

[38] E. U. Condon, G. H. Shortley, *Theory of Atomic Spectra* (Cambridge University Press, New York, 1951).

where we assume an equal g_c for all the core states, $g_l=0$ for a neutron and $g_s=-3.83$ nm (μ_0).

If we apply the decomposition theorem of the second kind (see Ref. 33), we get in units of μ_0:

$$\langle \tfrac{3}{2}\ \tfrac{3}{2}|\mu_z|\tfrac{3}{2}\ \tfrac{3}{2}\rangle=\tfrac{2}{5}\langle\tfrac{3}{2}\|\boldsymbol{\mu}\cdot\mathbf{J}\|\tfrac{3}{2}\rangle$$
$$=\tfrac{2}{5}\langle\tfrac{3}{2}\|g_c(\mathbf{L}^2+\mathbf{L}\cdot\mathbf{j})+g_s(\mathbf{s}\cdot\mathbf{L}+\mathbf{s}\cdot\mathbf{l}+\mathbf{s}^2)\|\tfrac{3}{2}\rangle. \quad (6.17)$$

Evaluation of this matrix element gives:

$$\langle\tfrac{3}{2}\ \tfrac{3}{2}|\mu_z|\tfrac{3}{2}\ \tfrac{3}{2}\rangle=0.1964g_s+0.5948g_c \quad (6.18a)$$

$$=0.2304g_s+0.5116g_c. \quad (6.18b)$$

To obtain the measured value of -0.662 nm we need a g_c of

$$g_c\approx 0.15 \text{ in case (a)}$$
$$g_c\approx 0.43 \text{ in case (b)}.$$

Bauer and Deutsch[39] have measured the g_c values of the first excited states of Sm^{152}, Gd^{154}, and Gd^{156}. They found that the values are (0.35±0.03), (0.367±0.03) and (0.32±0.03), which corresponds to a deviation from the Z/A values of 5–29%, if we consider the maximal errors. Our values show a deviation of 70% and 14%, respectively, from the Z/A value of 0.5 for Ne^{20}. While the second value is reasonable in comparison with the results in Ref. 39, the deviation of the first value is rather large.

If we assume that the wave functions for the Na^{21} ground state can be approximated by taking the Ne^{21} ground-state wave functions for a proton, we can give an estimate for the ft values and transition probabilities of the β decay between these two nuclei (see Sec. 2). The reduced transition probabilities for Fermi and Gamow–Teller interactions for allowed β^+ transitions are[40]

$$D_F(0)=\sum|\langle i|\mathbf{T}_-|f\rangle|^2$$
$$D_{GT}(0)=4\sum|\langle i|\mathbf{s}\cdot\mathbf{T}_-|f\rangle|^2, \quad (6.19)$$

where $\mathbf{s}$ is the spin operator of the particle and $\mathbf{T}_-$ is the component $\frac{1}{2}(T_1-iT_2)$ of the total isotopic spin. If we take the usual choice of the partial coupling constants as

$$g(1-x)^{1/2} \quad \text{and} \quad g(x)^{1/2} \quad (6.20)$$

for Fermi and Gamow–Teller transitions, respectively, the comparative half-lifes may be written in the form

$$ft=Bg[(1-x)D_F(0)+xD_{GT}(0)]^{-1}. \quad (6.21)$$

Conventional values are

$$Bg=2.6\times 10^3 \quad \text{and} \quad x=0.5. \quad (6.21a)$$

Evaluation of the matrix elements (6.19) gives:

For $\frac{3}{2}\rightarrow\frac{3}{2}$

$$D_F=1 \quad D_{GT}=0.354 \text{ (case a)}$$
$$D_F=1 \quad D_{GT}=0.256 \text{ (case b)}$$

For $\frac{3}{2}\rightarrow\frac{5}{2}$ (6.22)

$$D_F=0 \quad D_{GT}=0.053 \text{ (case a)}$$
$$D_F=0 \quad D_{GT}=0.049 \text{ (case b)}.$$

With these values we find that the relative probabilities of the transitions are

96% (97%) for $\frac{3}{2}^+\rightarrow\frac{3}{2}^+$ with a $\log ft$ value of 3.61 (3.58)

4% (3%) for $\frac{3}{2}^+\rightarrow\frac{5}{2}^+$ with a $\log ft$ value of 5.00 (5.02).

for cases (a), (b). These values compare favorably with the measured values given in Sec. 2.

7. DISCUSSION

(A) General

Basically the model contains three approximations: (1) It is assumed that the addition of an extra particle does not disturb the Ne^{20} core crucially. (2) The antisymmetrization between core and outside particles is not properly taken into account. (3) As the Ne^{20} core shows a deviation from the rotational pattern, the strong-coupling wave functions (5.8) are not the exact eigenfunctions of the Hamiltonian $H(1)=H_p+H_2(+H_4)$.

The validity of the first two approximations can only by supported by the results. The effect of the last approximation is to give too low $\frac{7}{2}$ and $\frac{9}{2}$ states and to imply the use of differing intrinsic quadrupole moments for the different core states, at least in the calculation of quadrupole features of Ne^{21}.

(B) Comparison of the Two Methods of Calculation

The SU_3 classification scheme for Ne^{21} implies a thorough mixture of five particle states with $l=0$, 2, while the model employed here treats the fifth particle in terms of an extreme single particle picture. So neither of the two methods of calculation can represent the situation obtained in the SU_3 scheme and we can only use it as a rough guide. While the first case (a) follows the single particle picture in assuming, that first the $j=\frac{5}{2}$ shell is filled independently (the same assumption is used in the Nilsson model), the second method (b) tries to simulate the SU_3 situation more closely at the cost of taking into consideration an additional term of the Hamiltonian in the form of a 2^4-2^4 pole interaction. As we do not know the exact form of the intrinsic core function and the appropriate coupling parameters f_2 and f_4, the estimates given in Sec. 5 are not too helpful.

If we compare the results, we find that in both cases the $\frac{3}{2}^+$ ground state contains approximately 99% of the $J=\frac{3}{2}$ state of the $j=\frac{5}{2}$, $K=\frac{3}{2}$ band, the $\frac{5}{2}^+$ first excited state contains approximately 85% of the $\frac{5}{2}$ state of this band, while the next well established state, the $\frac{1}{2}^+$ state at 2.79 MeV, shows a different composition in the two

[39] W. Bauer, M. Deutsch, Phys. Rev. 128, 751 (1962).
[40] A. Bohr, B. Mottelson, Kgl. Danske Videnskab. Selskab, Mat. Fys. Medd. 27, 118 (1957).

cases:

(a) 4% $j=\frac{3}{2}$, 96% $j=\frac{1}{2}$ (all $K=\frac{1}{2}$)

(b) 61% $j=\frac{5}{2}$, 21% $j=\frac{3}{2}$, 18% $j=\frac{1}{2}$ (all $K=\frac{1}{2}$).

Unfortunately, the $E2$ transition to the ground state is extremely weak:

$$\text{(a)} \quad B(E2;\tfrac{3}{2}\rightarrow\tfrac{1}{2})\approx 0.02\times10^{-52}\ \text{cm}^4$$

$$\text{(b)} \quad B(E2;\tfrac{3}{2}\rightarrow\tfrac{1}{2})\approx 0.2\times10^{-52}\ \text{cm}^4$$

(assuming equal $Q_{LL'}\approx 3.2\times10^{-25}$ cm^2), so the transition seems to be mainly $M1$. As the determination of the ratio of transition probabilities for $E2$ and $M1$ transitions is not too accurate, we have no possibility of discerning the two cases from this point so far. Furthermore by a choice of the $Q_{LL'}$ other than taking equal values [but within the limits given by Eq. (6.9) and Eq. (6.15)], we could obtain equal values of $B(E2;\frac{3}{2}\rightarrow\frac{1}{2})$ for the two cases. Similarly the $E2$ transition probability from the $\frac{1}{2}^+$ to the $\frac{5}{2}^+$ first excited state is small and in fact has not been detected. The quadrupole moment of the ground state, the $E2$ transition of the $\frac{3}{2}^+$ ground state to the $\frac{5}{2}^+$ first excited state and the β^+ decay features from Na^{21} are well reproduced by both methods. The magnetic moment of the ground state can be fitted better in case (b) than in case (a), though we have only an admixture of 0.2% of the $J=\frac{3}{2}$ $j=\frac{5}{2}$ $K=\frac{1}{2}$ state in the first case. A decreasing value of g_c for the core states with higher L instead of the uniform value employed in the calculation would improve the situation in case (a).

As we used positive intrinsic quadrupole moments to fit the quadrupole moment of the ground state, we used a negative value of the coupling strength f_2 in case (a), while it has to be taken positive in case (b). The first sign corresponds to the usual choice of the sign of the quadrupole-quadrupole force in collective model calculations.

Having only relatively scanty information about higher states in Ne^{21} and corresponding transitions, we come to the general conclusion, that we can not tell so far, which of the two modes of calculation gives a better picture of the actual situation. From the point of view of consistency one would favor the mode, which gives the smoothest variation of the parameters of the model, if we extend the calculation to neighboring nuclei in the same shell.

(C) Comparison with the Collective Model

In most cases in the $s-d$ shell the Nilsson model fails to give the right magnitude of the $E2$ transition probabilities and the quadrupole moment of the ground state, once the deformation parameter η is fixed to fit the spectrum and (or) the magnetic moment of the ground state (see Ref. 31). This suggests that the illustrative picture of the nucleus as an ellipsoid is not quite correct at least in this region of the Periodic Table.

The model employed here tries to avoid any classical picture with its consequences (particle j not a good quantum number in a cylindrical well). The Ne^{20} ground-state band shows approximately the features of a quantum-mechanical spherical rotator. This is a statement on the angular part of the core wave function only. It is conceivable, that the radial part of the core wave function gives the right magnitude of the measured intrinsic moments $Q_{LL'}$ by an appropriate coupling of the core particles. The coupling of this system with an ordinary shell-model particle gives strong-coupling wave functions as an intermediate step, if we use a body-fixed coordinate system. We find that the projection of the angular momentum of the particle on an intrinsic coordinate system K_j is a good quantum number. Here we had to assume, that the intrinsic core functions are the same for all the L states of the core. This condition is only approximately fulfilled in Ne^{20}.

In the further calculation the parameters of the coupling term of the Hamiltonian are treated essentially as free parameters, while all the core parameters C_L, g_c, $Q_{LL'}$, can (at least in principle) be obtained from experiment. So we are sure, that the core effects are properly taken into account.

The decoupling term of H_c [see Eq. (5.16)] is only a special case of the collective model form, as we are considering the coupling of one single particle only. As pointed out before (Sec. 3), the coupling term between particle spin and angular momentum of the core removes the resulting degeneracy in the case of a $j=\frac{1}{2}$ particle.

The choice of our parameters (5.23) corresponds to the usual Nilsson model parameters.

(a) $\eta=1.67$ $\kappa=0.135$ $\mu=0$ (with $f_2<0$)

(b) $\eta=3.18$ $\kappa=0.111$ $\mu=0$ (with $f_2>0$),

if we assume $\hbar\omega_0=41\ A^{-1/3}$ MeV$=14.86$ MeV. So we find that the spin-orbit splitting D is larger than the values $1.50\leq D\leq 3.00$ obtained from the usual choice of $0.05\leq\kappa\leq 0.10$. As $\hbar\omega_0$ decreases with increasing A, one would expect an increasing value of κ with the filling of the shell rather than a constant value. If Kurath's results for the variation of the parameter D in the p shell apply to the $s-d$ shell, one would even expect a rapid variation of κ in some part of the shell. The deformation parameter η in case (a), which corresponds more closely to the Nilsson model calculation, is smaller than the values used by Bhatt[31] ($\eta\approx 3, 4$) and Rakavy[28] [$\epsilon=0.48$ corresponding to $\eta(\kappa=0.05)=7$, $\eta(\kappa=0.10)=3.5$] for Ne^{21}.

In the model employed here we find that all the final Ne^{21} eigenfunctions of the lower states have an admixture of more than 80% of one particular $|J,j,K\rangle$ state, but still do not show any rotational structure. In fact as long as the core states are of the form $\chi_0 Y_{LM}$ and Q_2 is not equal to zero, we always obtain the strong coupling wave function (5.8) as an intermediate step, irrespective of the coupling strength of H_2.

ACKNOWLEDGMENT

The author would like to thank Professor D. C. Peaslee for the suggestion of the problem and many helpful discussions.